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World-Sheet Formulation of M Theory

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Abstract

We first review the interpretation of world-sheet defects as D branes described by a critical theory in 11 dimensions, that we interpret as M theory. We then show that D -brane recoil induces dynamically an anti-de-Sitter (AdS) space-time background, with criticality restored by a twelfth time-like dimension described by a Liouville field. Local physics in the bulk of this AdS_{11} may be described by an $osp(1|32, R) \otimes osp(1|32, R)$ topological gauge theory (TGT), with non-local boundary states in doubleton representations. We draw analogies with structures previously exhibited in two-dimensional black-hole models. Wilson loops of ‘matter’ in the TGT may be described by an effective string action, and defect condensation may yield string tension and cause a space-time metric to appear.

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1 Introduction

The recent exciting developments in the understanding of non-perturbative effects in the theory formerly known as strings [1] have led to tantalizing glimpses of a broader framework for the Theory of Everything (TOE), referred to variously as M or F theory. This TOE has been approached from several different perspectives. First were the strong-coupling limits of various string theories that had been thought distinct before the advent of duality. Consideration of Type IIA string led to M theory [2] and that of Type IIB string led to F theory. A second perspective has been provided by the low-energy limit. In the case of M theory, which is Lorentz invariant for non-trivial reasons, this low-energy limit leads to 11-dimensional $N = 1$ supergravity. The low-energy limit of F theory is less evident: it is not Lorentz invariant, and the relation to a candidate higher-dimensional supergravity theory is not yet clear. An interesting third perspective has been provided by Matrix theory [3], which proposes a non-perturbative formulation of M theory using light-cone quantization.

We have offered a fourth perspective [4], based on the world-sheet σ -model formulation of string theory. By extending the space of conformal field theories describing critical string theories to general renormalizable two-dimensional field theories, one is able to address issues in non-critical string theory [5, 6]. The world-sheet renormalization scale may be identified with a Liouville field [7] whose dynamics is non-trivial away from criticality. One example of an interesting world-sheet field theory is the non-compact Wess-Zumino model [8] that describes a black hole in $1+1$ dimensions, which may be formulated as a monopole defect on the world sheet [9]. We have recently shown that the supersymmetrization of this model is conformal in 11 dimensions [4], which we interpret as the world-sheet description of the massless solitons that appear in the strong-coupling string approach to M theory. This analysis is reviewed in more detail below, together with some indications that a twelfth dimension might be described by a Liouville field.

A fifth perspective on M theory has been the recent suggestion [10] that it might be equivalent at short distances to a Chern-Simons topological gauge theory (TGT) based on the supergroup $Osp(1|32, R) \otimes Osp(1|32, R)$. The idea that a TGT might underly quantum gravity has been recurrent in recent years. One such theory was found at the core of the $(1+1)$ -dimensional string black-hole model [11, 12], and it provides a natural incarnation of the holographic principle. The provocative proposal of Horava [10] raises several questions: is the choice of supergroup unique? what other degrees of freedom are present in M theory? what is the nature of the non-trivial dynamics that leads to the generation of space-time? and many others.

The purpose of this paper is to address these questions from the world-sheet perspective outlined above, building a bridge to the TGT perspective that also illuminates many aspects of the relationship between M theory and F theory. We recall that the appropriately supersymmetrized world-sheet monopole corresponds to a target-space D brane, and point out that its recoil induces an anti-de-Sitter (AdS) metric in the 11-dimensional target space, whose corresponding supergroup

must be at least as large as $Osp(1|32, R)$. Within this approach, there is an extra time-like dimension parametrized by a Liouville field. We point out that [13] the minimal supergroup extension of the Lorentz group in $11 + 1$ dimensions is $Osp(1|32, R) \otimes Osp(1|32, R)$, which is broken to $Osp(1|32, R)$ because of the Lorentz non-invariance of F theory. Horava's TGT is a local short-distance field theory with the $Osp(1|32, R) \otimes Osp(1|32, R)$ symmetry, but the full M -theory dynamics involves non-local structures that may be expressed as Wilson loops on the boundary of the 11-dimensional adS space. Singleton [14, 15, 16] and higher infinite-dimensional unitary representations of $Osp(1|32, R)$ describe non-local boundary states. We recall that supersymmetric Wilson loops have a string interpretation [17], in terms of which the world-sheet monopoles characterize defects at the interface with AdS space.

The layout of this paper is as follows. In section 2, we review aspects of our previous analysis of the two-dimensional string black-hole model that later find echos in our analysis of M theory. In section 3 we discuss the critical monopole and vortex deformations in superstring in 11 dimensions, and use D -brane recoil to derive in section 4 the corresponding AdS_{11} metric on target space-time. In section 5 we motivate the appearance of the $Osp(1|32, R) \otimes Osp(1|32, R)$ supergroup structure in the short-distance TGT, and in section 6 we draw parallels between AdS black holes and our earlier two-dimensional work. In section 7 we develop the interpretation of strings as Wilson loops in this TGT, and in section 8 we make some conjectures and advertize open issues within the perspective developed here.

2 Black Holes as World-Sheet Monopole Defects, and the Appearance of TGT at the Core

A relevant precursor of the discussion in this paper is found in the two-dimensional black-hole model. The Euclidean target-space version may be described in terms of a vortex defect on the world sheet [18, 19, 9], obtained as the solution X_v of the equation

$$\partial_z \bar{\partial}_z X_v = \frac{i\pi q_v}{2} [\delta(z - z_1) - \delta(z - z_2)] \quad (1)$$

where q_v is the vortex charge and $z_{1,2}$ are the locations of a vortex and antivortex, respectively, which we may map to the origin and the point at infinity. The corresponding solution to (1) is

$$X_v = q_v \text{Im} \ln z \quad (2)$$

and we see that the vortex charge q_v must be integer. To see that the solution (2) corresponds to a black hole, we introduce the space-time coordinates (r, θ) :

$$z \equiv (e^r - e^{-r})e^{i\theta} \quad (3)$$

in terms of which the induced target-space metric is

$$ds^2 = \frac{dzd\bar{z}}{1 + z\bar{z}} = dr^2 + \tanh^2 r d\theta^2 \quad (4)$$

which we recognize as a Euclidean black hole located at $r = 0$. We recall that this model can be regarded as an $SL(2, R)/U(1)$ Wess-Zumino coset model, with gauge field

$$A_z \rightarrow \epsilon^2 \partial_z \theta \quad (5)$$

as $r \equiv \epsilon \rightarrow 0$. We see that the gauge field is singular at the origin, so that the world-sheet defect may be interpreted as a monopole of the compact $U(1)$ gauge group.

There are related ‘spike’ configurations which are solutions of the equation

$$\partial_z \bar{\partial}_z X_m = -\frac{\pi q_m}{2} [\delta(z - z_1) - \delta(z - z_2)] \quad (6)$$

given by

$$X_m = q_m \text{Re} \ln z \quad (7)$$

It is easy to see that single-valuedness of the partition function imposes the following quantization condition:

$$2\pi\beta q_v q_m = \text{integer} \quad (8)$$

at finite temperature $T \equiv \beta^{-1} \neq 0$. Making the change of variables

$$|z|^2 \equiv -uv \quad : \quad u = e^{R+t}, v = -e^{R-t} \quad (9)$$

and identifying $R \equiv r + \ln(1 - e^{-r})$, we see that the ‘spike’ (7) corresponds to a Minkowski black hole [9]:

$$ds^2 = \frac{dzd\bar{z}}{1 + z\bar{z}} = -\frac{dudv}{1 - uv} = dr^2 - \tanh^2 r dt^2 \quad (10)$$

The coordinates u, v are natural in the Wess-Zumino description of the Minkowski black hole, which involves gauging the non-compact $O(1, 1)$ subgroup of $SL(2, R)$. Reparametrizing the neighbourhood of the singularity by $w \sim \ln u \sim -\ln v$, one finds a topological gauge theory (TGT) on the world sheet:

$$S_{CS} = i \int d^2 z \sqrt{h} \frac{k}{2\pi} w \epsilon^{ij} F(A)_{ij} + \dots \quad (11)$$

where h is the world-sheet metric, F is the field strength of the non-compact Abelian gauge field, and the dots represent additional ‘matter’ or ‘magnon’ fields. Their generic form close to the singularity is [8]

$$-\frac{k}{2\pi} \int d^2 z \sqrt{h} h^{ij} D_i a D_j b + \dots \quad (12)$$

where D_i is an $O(1, 1)$ covariant derivative and $ab + uv = 1$. We see from this that a non-zero condensate $\langle ab \rangle \neq 0$ corresponds to $\langle uv \rangle \neq 1$ and hence a non-trivial target space-time metric (10).

In this connection, we recall that there appears an enhanced symmetry at the singularity of the black hole [12], since the topological world-sheet theory describing

the singularity is characterized by a target $W_{1+\infty} \otimes W_{1+\infty}$ symmetry. When the space-time metric is generated away from the singularity, there is a spontaneous breaking of $W_{1+\infty} \otimes W_{1+\infty} \rightarrow W_{1+\infty}$ due to the expectation value $\langle ab \rangle \neq 0$, associated with Wilson loops surrounding the world-sheet defects.

This is a convenient point to preview the two key ways in which the two-dimensional black hole model described above is relevant to the construction of M theory. The world sheet may be mapped onto the target space-time in two dimensions, and this two-dimensional example may usefully be viewed from either perspective. On the world sheet, as we discuss in the next section, when the above monopole solution is supersymmetrized, it becomes a marginal deformation when the string is embedded in an 11-dimensional space-time. We have suggested previously that this limit corresponds to the masslessness of the D -brane representation of target-space black holes in the strong-coupling limit of M theory when it becomes 11-dimensional. The monopoles can be viewed as puncturing holes in the world sheet through which the core of the space-time theory can be visualized.

On the other hand, interpreting the two-dimensional model from the space-time point of view, we see at the core a TGT with a non-compact gauge group, analogous to that proposed by Horava [10]. The task we tackle in Section 4 is that of identifying the ‘matter’ fields that appear around the core by analogy with the fields a, b above, whose condensation generates the space-time metric. Before addressing these issues, though, we first examine more closely the supersymmetric world-sheet monopole model in an 11-dimensional space time.

3 Critical Defects in 11-Dimensional Superstring

The supersymmetrization of the above world-sheet defects may be represented using a sine-Gordon theory [19, 4] with local $n = 1$ supersymmetry, which has the following monopole deformation operator:

$$V_m = \bar{\psi}\psi : \cos\left[\frac{q_m}{\beta_{n=1}^{1/2}}(\phi(z) - \phi(\bar{z}))\right] : \quad (13)$$

where the $\psi, \bar{\psi}$ are world-sheet fermions with conformal dimensions $(1/2, 0), (0, 1/2)$ respectively, and ϕ is a Liouville field. The effective temperature $1/\beta_{n=1}$ is related to the matter central charge by [19]

$$\beta_{n=1} = \frac{2}{\pi(d-9)} \quad (14)$$

where we assume that $d > 9$. The corresponding vortex deformation operator is

$$V_v = \bar{\psi}\psi : \cos[2\pi q_v \beta_{n=1}^{1/2}(\phi(z) + \phi(\bar{z}))] : \quad (15)$$

where q_v is the vortex charge. Including the conformal dimensions of the fermion fields, we find that the conformal dimensions of the vortex and monopole operators are

$$\Delta_v = \frac{1}{2} + \frac{1}{2}\pi\beta_{n=1}q_v^2 = \frac{1}{2} + \frac{q_v^2}{(d-9)}, \quad \Delta_m = \frac{1}{2} + \frac{1}{8\pi\beta_{n=1}}q_m^2 = \frac{1}{2} + \frac{q_m^2(d-9)}{16} \quad (16)$$

respectively. We see that the supersymmetric vortex deformation with minimal charge $|q_v| = 1$ is marginal when the matter central charge $d = 11$. Below this limiting value, the vortex deformation is irrelevant. The quantization condition imposed by single-valuedness of the partition function tells us that the minimum allowed charge for a dual monopole defect is $|q_m| = (d-9)/4$, which is irrelevant for $14.04 > d > 11$. We therefore see that $d = 11$ is the critical dimension in which both the supersymmetric vortex and monopole deformations are marginal. On the world sheet, this corresponds to a Berezinskii-Kosterlitz-Thouless transition [20], with an unstable plasma phase of free vortex defects in $11 < d < 14.04$, whilst monopoles are bound for matter central charges $d > 14.04$. We note that $d = 11$ is the maximal dimensionality of space in which it is possible to have Lorentz-covariant local supersymmetric theories, whilst if one relaxes the requirement of Lorentz covariance a 12- (or higher-) dimensional target space may be allowed [21].

In terms of the ‘temperature’ (14), associated with the central-charge deficit of the matter theory, the above pattern of phases may be expressed as follows:

- (i) $T < T_{BKT-vortex}$, corresponding to $d < 11$: vortices bound, monopoles free,
- (ii) $T_{BKT-vortex} < T < T_{BKT-monop}$ corresponding to $11 < d < 14.04$: plasma of vortices and monopoles,
- (iii) $T > T_{BKT-monop}$ corresponding to $d > 14.04$: monopoles bound, vortices free.

where the two critical Berezinskii-Kosterlitz-Thouless temperatures are familiar from the two-dimensional XY model [20]. In our Liouville picture, these critical temperatures correspond to critical values of the central charge d , as explained above, whereas in critical strings such temperatures correspond to critical values of the radius of the compactified dimension [18].

We have argued that these defects correspond to D branes, since correlators involving defects and closed-string operators have cuts for generic values of $\Delta_{v,m}$. These cause the theory to become effectively that of an open string. One may then impose Dirichlet boundary conditions on the boundaries of the effective world sheet, i.e., along the cuts, obtaining solitonic D -brane configurations which become massless when $d \rightarrow 11$. The world-sheet Berezinskii-Kosterlitz-Thouless transition when $d = 11$ corresponds to the D -brane condensation that occurs in the strong-coupling limit of M theory. It is known that the low-energy limit of this critical theory is provided by 11-dimensional supergravity, which possesses only 3- and 5-brane solitonic solutions.

4 Anti-de-Sitter Space Time from D -Particle Recoil

In this section we discuss how eleven-dimensional anti-de-Sitter space time AdS_{11} arises from our Liouville approach to D -brane recoil [22]. The D brane is described as above by a world-sheet defect, whose interaction with a closed-string state is described by a pair of logarithmic deformations [23], corresponding to the collective coordinate y_i and velocity u_i of the recoiling D particle [24, 25]. Before the recoil, the world-sheet theory with defects is conformally invariant. However, the logarithmic operators are slightly relevant [24], in a world-sheet renormalization group sense, with anomalous dimension $\Delta = -\frac{\epsilon^2}{2}$, where ϵ is a regularization parameter specified below. Thus, the recoiling D particle is no longer described by a conformal theory on the world sheet. To restore conformal invariance, one has to invoke Liouville dressing [6], which increases the target space-time dimensionality to $d+1$. Because of the supercriticality [5] of the central charge d of the stringy σ model, which had been critical before including recoil effects, the Liouville field has Minkowski signature in this approach. In evaluating the σ -model path integral, it is convenient to work with a Euclidean time X^0 in the d -dimensional base space.¹ Thus we obtain an effective curved space-time manifold F in $d+1$ dimensions, with signature $(1, d)$, which is described [26] by a metric of the form:

$$G_{00} = -1, G_{ij} = \delta_{ij}, G_{0i} = G_{i0} = f_i(y_i, t) = \epsilon(\epsilon y_i + u_i t), \quad i, j = 1, \dots, d \quad (17)$$

where the regularization parameter $\epsilon \rightarrow 0^+$ is related [24] to the world-sheet size L via

$$\epsilon^{-2} \sim \eta \ln(L/a)^2, \quad (18)$$

where $\eta = -1$ for a Liouville mode t of Minkowski signature, and a a world-sheet short-distance cutoff. The quantities y_i and u_i represent the collective coordinates and velocity of a d -dimensional D(irichlet)-particle.

In our case, the original theory is conformal for $d = 11$, so the F manifold has twelve dimensions. The D particle is a point-like stringy soliton, with $d = 11$ collective coordinates satisfying Dirichlet boundary conditions on the open world sheet that appears in the presence of a defect. Thus the above Liouville theory describes a 12-dimensional space time with *two times* [21] if the basis space is taken to have Minkowski signature. The fact that the Liouville σ -model dilaton is linear in time reflects the non-covariant nature of the background [5]. This is consistent with the Lorentz-non-covariant formalism of 12-dimensional superstrings [21], which reflects the need to use null vectors to construct the appropriate supersymmetries. In our approach, this non-covariant nature is a natural consequence of the Liouville dressing, prior to supersymmetry, but this remark provides for a smooth supersymmetrization of the results.

¹Note that the time X^0 is therefore distinct from the Liouville time t , which necessarily has Minkowski signature. In the case of gauge theories, the Euclidean time X^0 may be thought of as temperature.

We recall [26] that the components of the Ricci tensor for the above 12-dimensional F manifold are:

$$R_{00} = -\frac{1}{(1 + \sum_{i=1}^d f_i^2)^2} \left(\sum_{i=1}^d f_i \frac{\partial f_i}{\partial t} \right) \left[\sum_{j=1}^d \frac{\partial f_j}{\partial y_j} \left(1 + \sum_{k=1, k \neq j}^d f_k^2 \right) \right] \quad (19)$$

$$+ \frac{1}{(1 + \sum_{i=1}^d f_i^2)} \left[\sum_{i=1}^d \frac{\partial^2 f_i}{\partial y_i \partial t} \left(1 + \sum_{j=1, j \neq i}^d f_j^2 \right) \right] \quad (20)$$

$$R_{ii} = \frac{1}{(1 + \sum_{k=1}^d f_k^2)^2} \left\{ \frac{\partial f_i}{\partial y_i} \left(\sum_{j=1}^d f_j \frac{\partial f_j}{\partial t} \right) - \left(1 + \sum_{k=1}^d f_k^2 \right) \frac{\partial^2 f_i}{\partial y_i \partial t} \right. \\ \left. + \frac{\partial f_i}{\partial y_i} \left[\sum_{j=1, j \neq i}^d \frac{\partial f_j}{\partial y_j} \left(1 + \sum_{k=1, k \neq j}^d f_k^2 \right) \right] \right\} \quad (21)$$

$$R_{0i} = \frac{f_i}{(1 + \sum_{k=1}^d f_k^2)^2} \left\{ \frac{\partial f_i}{\partial y_i} \left(\sum_{j=1}^d f_j \frac{\partial f_j}{\partial t} \right) - \left(1 + \sum_{k=1}^d f_k^2 \right) \frac{\partial^2 f_i}{\partial y_i \partial t} \right\} \quad (22)$$

$$R_{ij} = \frac{1}{(1 + \sum_{k=1}^d f_k^2)^2} f_i f_j \frac{\partial f_i}{\partial y_i} \frac{\partial f_j}{\partial y_j} \quad (23)$$

We consider below the asymptotic limit: $t \gg 0$. Moreover, we restrict ourselves to the limit where the recoil velocity $u_i \rightarrow 0$, which is encountered if the D particle is very heavy, with mass $M \propto 1/g_s$, where $g_s \rightarrow 0$ is the dual string coupling. In such a case the closed string state splits into two open ones ‘trapped’ on the D -particle defect. The collective coordinates of the latter exhibit quantum fluctuations of order [22] $\Delta y_i \sim |\epsilon^2 y_i|$. From the world-sheet point of view [9], this case of a very heavy D particle corresponds to a strongly-coupled defect, since the coupling e of the world-sheet defect is related to the dual string coupling g_s by

$$e\sqrt{\pi/3} \propto \frac{1}{\sqrt{g_s}} \quad (24)$$

corresponding to a world-sheet/target-space strong/weak coupling duality.

In the limit $u_i \rightarrow 0$, (23) implies that the only non-vanishing components of the Ricci tensor are:

$$R_{ii} \simeq \frac{\partial f_i}{\partial y_i} \left(\sum_{j=1; j \neq i}^d \frac{\partial f_j}{\partial y_j} [1 + \mathcal{O}(\epsilon^4)] \right) \frac{1}{(1 + \sum_{k=1}^d f_k^2)^2} \simeq \frac{-(d-1)/|\epsilon|^4}{(\frac{1}{|\epsilon|^4} - \sum_{k=1}^{d=11} |y_k|^2)^2} + \mathcal{O}(\epsilon^8) \quad (25)$$

where we have taken into account (18) and the Minkowski signature of the Liouville mode t . Thus, in this limiting case and for large $t \gg 0$, the Liouville mode decouples from the residual d -dimensional manifold. We may write (25) as

$$R_{ij} = \mathcal{G}_{ij} R \quad (26)$$

where \mathcal{G}_{ij} is a dimensionless diagonal metric, corresponding to the line element:

$$ds^2 = \frac{|\epsilon|^{-4} \sum_{i=1}^d dy_i^2}{\left(\frac{1}{|\epsilon|^4} - \sum_{i=1}^d |y_i|^2\right)^2} \quad (27)$$

This metric describes the *interior* of a d -dimensional ball, which is the Euclideanized version of an AdS space time [27]. One can easily check that the curvature of the Minkowski version of (27) is *constant* and *negative*: $R = -4d(d-1)|\epsilon|^4$, independent of the exact location of the D brane.

It is interesting that the metric of the space time (27) exhibits a coordinate singularity at $\sum_{i=1}^d |y_i|^2 = |\epsilon|^{-4}$, which prevents a naive extension of the *open* ball B^d to the *closed* ball \overline{B}^d , including the boundary sphere S^{d-1} . The metric that extends to \overline{B}^d is provided by a conformal transformation of (27) [27]:

$$d\tilde{s}^2 = \mathcal{F}^2 ds^2 \quad (28)$$

Choosing, say, $\mathcal{F} = |\epsilon|^{-4} - \sum_{i=1}^d |y_i|^2$ results in $d\tilde{s}^2$ being associated with the metric on a sphere S^{d-1} of radius $\frac{1}{|\epsilon|^2}$. In general, \mathcal{F} may be changed by any conformal transformation, leading to a conformally invariant Euclidean S^{d-1} space as the boundary of an AdS_d space time, whose metric is invariant under the Lorentz group $SO(1, d)$.

We close this section by recalling that AdS space time is the only type of constant-curvature background, for space time dimensionality $d > 2$, which is consistent with local supersymmetry [28]. This is of vital importance here, because D branes are stable *only* in superstring theories.

5 Specification of the Local Theory in the AdS_{11} Bulk

The important property of AdS space times for the next part of our discussion is the existence of powerful theorems implying that, if a classical field theory is specified in the boundary of the AdS space, then it has a *unique* extension to the bulk [27, 29]. These theorems underly the *holographic* nature of field/string theories in AdS space times, in the sense that all the information about the bulk AdS theory is *encoded* in the boundary theory [30, 27].

The question now arises, what is the bulk AdS_{11} theory underlying M theory? Some clues are provided by the construction of AdS_{11} given in the previous section. It is natural to look for a theory that becomes local in the short-distance limit. This theory should, moreover, have a local symmetry that arises naturally in such a framework. The natural candidates are gauge theory and gravity, combined of course with supersymmetry. In the case of gauge theory, since a conventional quadratic kinematic term is irrelevant by simple dimensional counting in more than four space-time dimensions, the most likely possibility is a Topological Gauge Theory (TGT) of Chern-Simons type. Furthermore, since it is known in principle how supergravity

may arise from an underlying TGT [31], this may also provide a framework in which the notion of a space-time metric emerges dynamically. The remaining issue is the choice of gauge supergroup, and this is where the above construction of the AdS_{11} target space-time geometry can provide key input into the short-distance formulation of M theory as a supersymmetric TGT.

The minimal supergroup that incorporates the space-time symmetry of AdS_{11} is $Osp(1|32, R)$, so this should be a sub-supergroup of the conjectured gauge supergroup. However, our world-sheet approach provides a hint that the full gauge supergroup should be larger than this. We recall that the departure from criticality inherent to the non-trivial D -brane recoil that generates AdS_{11} could be absorbed by introducing a time-like Liouville field. This leads to an underlying $(2, 10)$ space-time signature, or $(1, 11)$ in the Euclideanized version needed for an adequate definition of the path integral. We also recall that there is an isomorphism between the minimal supergroup extension of the Lorentz group $SO(1, 11)$ and $Osp(1|32, R) \otimes Osp(1|32, R)$ [13]. It is therefore natural to propose that this may play a rôle in the supergroup of the local TGT. Note, however, that the Liouville field decouples in the conformal limit of zero recoil, and that the background field is linear, so any underlying $SO(1, 11)$ or $Osp(1|32, R) \otimes Osp(1|32, R)$ symmetry must be at least spontaneously broken. The natural minimal possibility is a $Osp(1|32, R) \otimes Osp(1|32, R) \rightarrow Osp(1|32, R)$ symmetry-breaking pattern, with the breaking accompanied by the appearance of an AdS_{11} metric at the world-sheet Berezinskii-Kosterlitz-Thouless transition point.

This is similar to the symmetry-breaking pattern proposed by [10]. In that analysis, the particular gauge supergroup $Osp(1|32, R) \otimes Osp(1|32, R)$ arose as the minimal supersymmetric extension of $Osp(1|32, R)$ with 64 supercharges [13, 32], which are necessary in M theory to ensure *parity invariance* and in order to obtain a consistent compactification of M theory to a heterotic string with the gauge group $E_8 \otimes E_8$ [33]. In our Liouville approach, such a group arises independently and naturally from the presence of the ‘auxiliary’ Liouville field.

6 Relation to Two-Dimensional Structures

It is known from the analysis in [16] that the contraction of $Osp(1|32, R) \otimes Osp(1|32, R)$ with the Poincare symmetry in 11-dimensional space time leads to a single diagonal $Osp(1|32, R)$ ².

Following [15], we now recall that $Osp(1|32, R)$ has a two-dimensional maximal sub-supergroup $Osp(16/2, R)$, which has been argued to capture the dynamics of $D0$ particles in the matrix-model approach to M -theory [3]. The maximal even subgroup of $Osp(16/2, R)$ is in turn $Sp(2, R) \otimes SO(16)$. It was argued in [15] that the factor $Sp(2, R)$ corresponds to an AdS_2 extension of the Poincare group in the longitudinal directions of the matrix D -brane theory. The connection to this formulation of $D0$

²The two factors correspond to different spinor representations of the $SO(1, 11)$ algebra [15, 16].

particles supports the motivation for the D -brane interpretation of the world-sheet defects and the recoil calculation presented earlier.

The singleton representations of $Sp(2, R)$, which live on the boundary of AdS_2 , when expanded in a ‘particle basis’, consist of an infinite tower of discrete-momentum states with ever-increasing quantized $U(1)$ eigenvalues [32]. Such an infinite tower of states was identified in [15] with the infinite tower of $D0$ branes with quantized longitudinal momentum that appear in matrix theory in the infinite-momentum frame [3]. This is consistent with the conjecture [30] that the conformally invariant field theory of the singleton representation on the boundary of AdS_2 is associated with an $N = 16$ $U(n \rightarrow \infty)$ Yang-Mills quantum-mechanical theory [15], which in turn describes matrix theory in the infinite-momentum frame.

We observe here that $Sp(2, R)$ is isomorphic to $SO(2, 1)$, as well as to AdS_2 . This may be related to the possibility of associating two-dimensional space times with three-dimensional Chern-Simons theories, whose dimensional reduction leads to AdS_2 . This possibility recalls that of two-dimensional stringy black-hole space times [9], as briefly reviewed in section 2. The appearance of a D -particle space time $AdS_2 \otimes O(16)$ through the ‘breaking’ (described by the contraction with Poincare symmetry) of $Osp(1/32, R) \otimes Osp(1/32, R)$ parallels the breaking of $W_{1+\infty} \otimes W_{1+\infty} \rightarrow W_{1+\infty}$ in that case. In the analysis of [9], the association of the two-dimensional model with a three-dimensional Chern-Simons theory with CP^1 ‘magnon’ fields a, b that represent matter away from the black-hole singularity, leads to an interesting symmetry-breaking pattern. A renormalization-group analysis has shown that space time appears as a non-trivial (infrared) fixed point of the flow. We present in the next section an alternative formulation of the appearance of a space-time metric in the full M theory.

The appearance of $D0$ particles provides a nice consistency check of the approach we used in section 4, employing Liouville D -particle recoil to obtain AdS_{11} dynamically. We now observe that AdS_2 structures can be associated with topology change in AdS_{11} . To see this, we first review briefly the relevant properties of AdS space times [34, 27]. For concreteness, we describe explicitly the AdS_4 case of [34]. The generalization to AdS_d , for general d , is straightforward [27]. The Minkowski-signature AdS Schwarzschild black hole solution of [34] corresponds to a metric line element of the form:

$$ds^2 = -V(dt)^2 + V^{-1}(dr)^2 + r^2 d\Omega^2 \quad (29)$$

where $d\Omega^2$ is the line element on a round two-sphere, r the radial coordinate of the AdS space, and t is the time coordinate. The Euclidean version of the space time has the topology $X_2 = B^2 \otimes S^{n-1}$, where $n = 3$ for [34].

According to the analysis of [34], there are two relevant critical temperatures in the AdS black-hole system:

- (i) the specific heat of a gas of black holes changes sign at the lowest critical temperature T_0 . For $T < T_0$ there is only radiation, and the topology of the finite-temperature space time is $X_1 = B^n \otimes S^1$, where $n = 3$ in [34].
- (ii) At temperatures above T_0 , the topology of the space time changes to include

black holes, becoming $X_2 = B^2 \otimes S^{n-1}$.

(iv) For temperatures greater than a higher value $T > T_2$, there is no equilibrium configuration without a black hole.³

The extension of the above analysis to our 11-dimensional AdS_{11} case is straightforward [27]. The important point to notice, for our purpose, is the fact that the black-hole space time X_2 includes the two-dimensional AdS_2 space, on the boundary of which live the quantum-mechanical $D0$ particles, as discussed in [15] and mentioned above. Therefore, we associate the group-theoretic observations of [15], with the topology change: $X_1 \rightarrow X_2$ due to the appearance of black-hole AdS space times. As discussed above, such topology changes may be interpreted as corresponding to condensation of world-sheet vortex defects. Notice, however, that the rôle of temperature in this case is played by the central charge deficit of the ‘matter’ theory (14). Thus, when the matter central charge d reaches the critical value, corresponding to the lowest temperature T_0 of [34], the topology changes, in the sense that the vacuum becomes dominated by an unstable plasma of free vortex and monopole world-sheet defects.

The presence of a discrete tower of states living on the boundary of AdS_2 has been argued [15] to be crucial for yielding the correct description of the $D0$ -particle quantum mechanics. Such delocalized states are therefore viewed as gravitational degrees of freedom. In this respect, the attention of the alert reader should be called to a parallel phenomenon that arises in two-dimensional stringy black holes with matter. There, conformal invariance on the world sheet requires, in a black-hole space time, the coupling to lowest-level zero-momentum string ‘tachyon’ states of discrete solitonic delocalized states that belong to higher string levels [36]. In the AdS_2 case, the discrete tower of states corresponds to a generalization of these lowest zero-momentum ‘tachyon’ states, rather than the higher-spin delocalized states of the two-dimensional black hole.

A final topic of relevance here is the tensoring of the singleton representation in each factor of $OSP(1|32, R)$. The resulting theory consists, according to [16], of a doubleton representation that lives on the boundary of AdS_{11} , which is a ten-dimensional Minkowski space, and is scale invariant. The next question concerns the coupling of such scale-invariant theories to superstrings, which is discussed in the next section.

7 Strings as Wilson Loops in TGTs

Invoking some mean-field conjectures, Horava [10] has shown how one can derive the field content of the 11-dimensional supergravity by calculating Wilson

³There is also an intermediate temperature T_1 : $T_0 < T < T_1 < T_2$, below which the free energy of the black hole is positive, so the black hole tends to evaporate, and above which the free energy of the configuration with the black hole and thermal radiation is lower than the corresponding configuration with just thermal radiation, so that the radiation tunnels to a black-hole state. In our approach, this tunnelling may be described by world-sheet instantons [7, 35], which we do not explore further here.

loops $\langle W(C) \rangle$ of ‘partons’ in his 11-dimensional TGT with gauge supergroup $Osp(1|32, R) \otimes Osp(1|32, R)$. Horava [10] did not provide a dynamical model for these ‘partons’ and Wilson loops. However, our Liouville string approach provides a natural conjecture for their origins, and, as we shall see below, a rather modified proposal for a local field-theory description of M theory.

As a prelude to our results, we first review briefly the work of [17], according to which certain scale-invariant Green-Schwarz superstring theories in flat target space are equivalent to Abelian gauge theories. This equivalence should be understood in the sense that

$$\langle W(C) \rangle \sim e^{iS_\sigma} \quad (30)$$

where S_σ is a world-sheet action that encodes the area of the Wilson loop, and $W(C)$ is some combination of observables, expressed in terms of chiral currents $e^{\int J.A}$, which go beyond the standard Wilson loops. The action S becomes a standard world-sheet action if a string scale is generated dynamically by an appropriate condensation mechanism.

The analysis of [17] was performed for four-dimensional target spaces, but it can be generalized straightforwardly to six and ten dimensions. For reasons of concreteness and calculational simplicity, we review the analysis in the four-dimensional case, where the gauge field theory coupling is dimensionless.

We consider an Abelian supersymmetric gauge theory, described by a standard Maxwell Lagrangian in a superfield form.⁴ The connection with string theory emerges by considering the Stokes theorem on a two-dimensional surface Σ , whose boundary is the loop C . If one parametrizes the curve C by λ , then one may write the exponent of the Wilson loop as

$$S_{int} = ie \int_C d\tau A(X(\tau)) \frac{\partial}{\partial \tau} X(\tau) \quad (31)$$

and the Stokes theorem tells us that

$$S_{int} = \frac{ie}{2} \int_{\Sigma(C)} d^2\sigma \epsilon^{ab} F_{ab}, \quad a, b = 1, 2, \quad (32)$$

where X^M , $M = 1, \dots, D$, is a D -dimensional space-time coordinate for the gauge theory. We denote by lower-case Latin indices the two-dimensional coordinates of the surface Σ , which plays the rôle of the world-sheet of the string and is equivalent to the gauge theory in question [37]. The quantity $F_{ab} = \partial_a X^M \partial_b X^N F_{NM} = \partial_a A_b - \partial_b A_a$ is the pull-back of the Maxwell tensor on the world sheet Σ , with A_a the corresponding projection of the gauge field on Σ :

$$A_a = v_a^M A_M \quad ; \quad v_a^M \equiv \partial_a X^M, a = 1, 2 ; M = 1, \dots, D (= 4). \quad (33)$$

From a two-dimensional view-point, this looks like a Chern-Simons term for a two-dimensional gauge theory on Σ , bounded by the loop C . The world sheet ‘magnetic

⁴The extension of this analysis to the non-Abelian case raises interesting technical issues that are currently under study.

field' corresponding to (33) reads:

$$\mathcal{B} = \epsilon^{\alpha\beta} \partial_\alpha A_\beta = \epsilon^{\alpha\beta} \partial_\alpha \partial_\beta X^M A(X)_M + \frac{1}{2} \epsilon^{\alpha\beta} \partial_\alpha X^M \partial_\beta X^N F_{NM} \quad (34)$$

Notice that the presence of world-sheet vortices is associated with the first term on the right-hand-side of (34), whilst world-sheet monopoles are associated with the second term, which is also gauge invariant in target space.

The novel observation of [17] is the possibility, in a supersymmetric gauge theory, of constructing a second superstring-like observable, in addition to the Wilson loop, which is again defined on the two-surface Σ , and is consistent with all the symmetries of the theory. The second observable is easily understood in the two-dimensional superfield formalism:

$$Z^{\mathcal{A}} \equiv (X^M, \theta^m, \theta^{\dot{m}}) \quad (35)$$

The pull-back basis v_a^M in (33) is now extended to $v_a^{\mathcal{A}} = E_{\mathcal{B}}^A \partial_a z^{\mathcal{B}}$, with the following components [17]:

$$\begin{aligned} v_a^{\alpha\dot{\alpha}} &= \partial_z x^{\alpha\dot{\alpha}} - \frac{i}{2} \left(\theta^\alpha(\sigma) \partial_a \theta^{\dot{\alpha}}(\sigma) + \theta^{\dot{\alpha}}(\sigma) \partial_a \theta^\alpha(\sigma) \right) \\ v_a^\alpha &= \partial_a \theta^\alpha(\sigma) \\ v_a^{\dot{\alpha}} &= \partial_a \theta^{\dot{\alpha}}(\sigma) \end{aligned} \quad (36)$$

in standard notation [38], where Greek dotted and undotted indices denote super-space components, with $x^{\alpha\dot{\alpha}} \equiv X^M$, etc.. Following [17], we now define

$$\begin{aligned} C_{ab}^{\alpha\beta} &\equiv \frac{i}{2} v_{a\dot{\beta}}^{(\alpha} v_b^{\beta)\dot{\beta}}, \\ C_{ab}^\alpha &\equiv v_a^{\alpha\dot{\alpha}} v_{b\dot{\alpha}} : \\ C^\alpha &= \epsilon^{ab} C_{ab}^\alpha, \quad C^{\alpha\beta} = \epsilon^{ab} C_{ab}^{\alpha\beta} \end{aligned} \quad (37)$$

with similar relations holding for appropriately-defined dotted components of C , as found in [17]. Note that C^α vanishes in the absence of supersymmetry, whilst $C^{\alpha\beta}$ exists also in non-supersymmetric gauge theories. In the presence of defects, as we shall later, this is no longer the case, and C^α can have non-supersymmetric remnants.

The supersymmetric Wilson loop, expressing the interaction between the superparticle and a supersymmetric gauge theory in the approach of [17], may now be expressed as:

$$\begin{aligned} W(C) &= e^{S_{int}^{(1)}}, \quad S_{int}^{(1)} \equiv \frac{i}{2} \int_{\Sigma(C)} d^2\sigma \epsilon^{ab} \mathcal{F}_{ab} \\ \mathcal{F}_{ab} &\equiv \epsilon_{ab} \left\{ \frac{1}{2} C^{\alpha\beta}(\sigma) D_\alpha W_\beta(x(\sigma), \theta(\sigma)) + \right. \\ &\quad \left. C^\alpha(\sigma) W_\alpha(x(\sigma), \theta(\sigma)) + h.c. \right\} \end{aligned} \quad (38)$$

where $W_\alpha(x(\sigma), \theta(\sigma))$ is the chiral superfield of the supersymmetric Abelian gauge theory. The exponent $S_{int}^{(1)}$ clearly reduces to the standard expression (31) in non-supersymmetric cases.

It was pointed out in [17] that there is a second superstring-like observable, $\Psi(\Sigma)$, defined on the world-sheet surface Σ , which is constructed out of the C_{ab}^α components, which therefore - in the absence of world-sheet defects - exists only in supersymmetric gauge theories:

$$\Psi(\Sigma) \equiv e^{iS_{int}^{(2)}}, \quad S_{int}^{(2)} \equiv \kappa \int_{\Sigma(C)} d^2\sigma \sqrt{-\gamma} \gamma^{ab} C_{ab}^\alpha(\sigma) W_\alpha(x(\sigma), \theta(\sigma)) + h.c. \quad (39)$$

where γ^{ab} is the metric on Σ . This term, unlike the standard Wilson loop, is not a total world-sheet derivative, and therefore lives in the bulk of the world-sheet Σ , and depends on the metric γ . The coupling constant κ is defined classically as an independent coupling. However, one expects that quantum effects will relate it to the gauge coupling constant e , a point we return to later. This second observable has been expressed in [17] in terms of ‘chiral’ currents on Σ , located at the string source:

$$\begin{aligned} S_{int}^{(2)} &= \int d^6Z (\mathcal{J}^\alpha W_\alpha + h.c.), \\ \mathcal{J}^\alpha &\equiv \kappa \int_{\Sigma(C)} d^2\sigma \sqrt{-\gamma} \gamma^{ab} C_{ab}^\alpha(\sigma) \delta^{(6)}(Z - Z(\sigma)), \\ \delta^{(6)}(Z - Z(\sigma)) &= \delta^{(4)}(Z - Z(\sigma)) (\theta - \theta(\sigma))^2 \end{aligned} \quad (40)$$

Upon integrating out the gauge field components in (39), i.e., considering the vacuum expectation value $\langle \Psi(\Sigma) \rangle$, where $\langle \dots \rangle$ denotes averaging with respect to the Maxwell action for the gauge field, the authors of [17] have obtained a superstring-like action, which is scale invariant in target space, as well as on the world sheet:

$$\begin{aligned} \langle \Psi(\sigma) \rangle_{Maxwell} &= e^{S_0 + S_1} \\ S_0 &\equiv \frac{\kappa_0^2}{16\pi} \int_{\Sigma} d^2\sigma \sqrt{-\gamma} \gamma^{ab} v_a^M v_b^N \sigma_M \sigma_N, \\ S_1 &\equiv \frac{\kappa_1^2}{4\pi} \int_{\Sigma(C)} \sqrt{-\gamma} \gamma^{ab} v_a^M v_b^N \eta_{MN} \sigma^K \sigma_K \end{aligned} \quad (41)$$

where upper-case Latin indices denote target-space indices ($M, N, K = 1, \dots, d(=4)$), and

$$\begin{aligned} v_a^M &\equiv \partial_a X^M(\sigma) - i\bar{\theta}^m(\sigma) \Gamma^M \partial_a \theta_m(\sigma) \\ \sigma^M &= \frac{\sqrt{-\gamma} \epsilon^{ab}}{\sqrt{\det G}} \partial_a v_b^M, \quad G_{ab} \equiv v_a^M v_b^N \eta_{MN} \end{aligned} \quad (42)$$

in standard four-component notation in target space, where the m are spinor indices, and the Γ^M are four-dimensional Dirac matrices. The dimensionless coupling constants $\kappa_{0,1}$ appear arbitrary at the classical level, but one expects them to be related to the dimensionless gauge coupling e , in the quantum theory.

The important observation in [17] was the fact that the world-sheet action S_2 (39) resembles the classical Green-Schwarz superstring action in flat four-dimensional target space, provided that condensation occurs for the composite field

$$\Phi \equiv \sigma^M \sigma_M, \quad M = 1, \dots, D(=4), \quad (43)$$

which may be interpreted as the dilaton in target space, so that

$$\frac{\kappa_1^2}{4\pi} < \Phi > = \mu_{string \text{ tension}} \quad (44)$$

Such condensation would enable the string tension μ to be obtained from a gauge theory without dimensionful parameters. The question of physical interest is what causes this condensation, which presumably is responsible for a ‘spontaneous breaking’ of the scale invariance of four-dimensional string theory.

We note that the above definition of the composite field Φ may be extended to higher-dimensional cases, which are of interest to us in this M -theory application, by simply contracting the integrand of (39) with appropriate powers of Φ so as to make the coupling constant κ dimensionless. This can be understood as the definition of the superstring observable Ψ in more than four target space-time dimensions. It is important to note that the dimensionality of the coupling constant κ is the same as that of the gauge theory coupling e only for space-time dimensionality four, six and ten, where supersymmetric theories exist as well.

We now present a scenario for the formation of such a condensate, which is an extension of the ideas of [27]. We associate the second observable (39) of [17] to the condensation of non-trivial world-sheet defects. The scenario is based on the recent interpretation [27] of confinement in purely gluonic non-Abelian gauge theories, by means of a holographic principle encoding confinement quantum physics in the classical geometry of uncompactified AdS_5 space times. In such a picture, four-dimensional conformally-invariant Minkowski space time is viewed as the boundary of an AdS_5 space time, in the spirit of [30, 27]. Macroscopic black holes disappear at temperatures below T_0 , where only radiation-dominated universes exist [34], as discussed above. Witten [27], has associated this temperature to a confining-deconfining phase transition for quarks, and stressed the fact that, for spatial Wilson loops the area law and the associated string tension are obtained only for finite-temperature field theory, because of the conformal invariance of the zero-temperature four-dimensional gauge theory, whose renormalization-group β function vanishes identically.

It is important to notice that condensation of composite operators can occur for both vortex (2) and monopole (7) configurations. Let us first examine the case of the manifold with topology $X_1 = B^n \otimes S^1$. In this case, one may consider the role of vortices on the world sheet of the string, wrapped around the compact dimension S^1 . The condensation of such vortices, bound into pairs with antivortices, results in the quantity Φ (43) acquiring a non-trivial vacuum expectation value $< \Phi > \neq 0$. In the classical AdS picture described above, it is the non-trivial Planckian dynamics of a five-dimensional space time which is responsible for the above phenomenon. In such a case the standard Berezinskii-Kosterlitz-Thouless phase transition temperature for vortex condensation on the world sheet of critical strings [18] may be identified with T_0 of the AdS Black Hole space time.

8 A Conjecture for the Structure and Dynamics of M Theory

We are now equipped to formulate a conjecture on the structure and dynamics of M theory. Since ten-dimensional Minkowski space time may be considered as the boundary of AdS_{11} , which arises dynamically through the Liouville dressing of a quantum-fluctuating D particle, we invoke the analysis of [17] for Abelian gauge theories, to conjecture that there exists a conformal field theory on the ten-dimensional Minkowski space time \mathcal{M} , which is dual to AdS_{11} in the following sense [30]:

$$\langle e^{\int_{\mathcal{M}} \phi_0 \mathcal{O}} \rangle_{CFT} = Z_{\text{AdS}_{11}}(\phi) \quad (45)$$

where the \mathcal{O} are appropriate local operators of the conformal field theory. This is supported by the fact that the conformal group of \mathcal{M} is the same as $SO(2, 10)$. Gunaydin has suggested [16] that the conformal field theory might be the doubleton field theory living on \mathcal{M} .

At this point, we should also remark that, according to Horava [10], the correspondence with the 11-dimensional supergravity occurs through partons of the gauge group $OSP(1/32, R) \otimes OSP(1/32, R)$ in an 11-dimensional Chern Simons topological Gauge theory. This is different from the conjecture (45), which seems more natural from the point of view of [17, 30]. However, the approach of [10] may be connected to the conjecture (45), if one makes a connection between the Chern-Simons gauge theory on $OSP(1/32, R) \otimes OSP(1/32, R)$ used in [10] with an appropriate string theory in a gravitational background in 12 space-time dimensions. Such a correspondence should be expected from the isomorphism of the supergroup extension of $SO(1, 11)$ with $OSP(1/32, R) \otimes OSP(1/32, R)$. Summing this string theory, of Liouville type, over world-sheet genera in gravitational 12-dimensional backgrounds, as discussed above, produces a path integral over string backgrounds, as discussed in [39].

The fact that the 12-dimensional space-time backgrounds are not Lorentz covariant is not a problem, in view of the non-covariant form of the Liouville theory. The basic formula of [39] for such a string-theory space quantization may be summarized as:

$$\mathcal{Z} = \int Dg^i e^{-C[g^j] + \int d^2\sigma \partial_\alpha X^M \partial^\alpha X^N \langle \partial^\gamma J_{\gamma, M} \partial^\delta J_{\delta, N} \rangle_g} + \dots \quad (46)$$

where the $\{g^i\}$ denotes an appropriate set of backgrounds, including the graviton, the \dots denote antisymmetric tensor and other string sources, $C[g]$ is the Zamolodchikov central-charge action, which plays the rôle of an effective (low-energy) target-space action of the string theory, and the J_M are Noether currents, corresponding to the translation invariance in target space time of the string. The measure of integration Dg^i arises from summing over world-sheet genera.

It was argued in [39] that condensation in the two-point function of the currents may lead to well-defined normalizable metric backgrounds, coupled to the string source, provided that the currents are logarithmic [23]. The same would be true for antisymmetric tensor source terms, etc.. This means that there are appropriate p -

brane solutions, obtainable as saddle points, with respect to the various backgrounds \bar{g}^i , of the partition function (46). For instance, for graviton backgrounds,

$$\frac{\delta}{\delta G_{MN}} C[\bar{g}] + V_{MN} = 0 \quad (47)$$

where the graviton vertex operator V_{MN} plays the rôle of an external string source.

Among such solutions there would be AdS backgrounds, such as the ones discussed above, which are already known to be associated with logarithmic recoil operators [24]. Based on these considerations, one may expect a connection of the quantum theory of [10]: $\langle W(C) \rangle \sim \mathcal{Z}$, with the classical AdS theory, c.f. the mean-field approximation in (47). In that case, a classical AdS effective action which has a holographic property [27, 30] may be viewed as the mean-field result of the quantum theory of the fluctuating stringy background (46). In this picture, the appearance of a doubleton conformal field theory (45) on the boundary Minkowski space time may well be valid ⁵, thereby unifying the conjecture (45), also made in [16], with that made in [10]. However, we re-emphasize that the above discussion should be considered conjectural at the present stage.

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⁵In the sense that, at least at present, we do not see an apparent conflict between the two conjectures.

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